Mining Multi-level Association Rules in Large Databases

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Overview

- What is MLAR?
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- Concepts behind the Method
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- The Method For Mining Multi-Level Association Rules
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- Conclusions and Future Work
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- Conclusions and Future Work
- Exam Questions
What is MLAR Overview

- MLAR stands for Multi-Level Association Rule
- Motivation for MLDM*
- Requirements for MLDM*

*MLDM: Multi Level Data Mining
What’s different between each of these rules?

- **Rule A**: 70% of customers who bought diapers also bought beer
- **Rule B**: 45% of customers who bought cloth diapers also bought dark beer
- **Rule C**: 35% of customers who bought pampers also bought Samuel Adams
This process is called Drilling down

- **Rule A** applies at a *generic* higher level of abstraction (product)
- **Rule B** applies at a *more specific* level of abstraction (category)
- **Rule C** applies at the *lowest* level of abstraction (brand)
Why Drill Down?

- The information is more valuable
- Different levels of associations enable different strategies for marketing
Why Drill Down?

- Remove uninteresting rules
- toy $\implies$ milk is not interesting (coincidence)
In a Nutshell

We need to be able to create *interesting* and *valuable* rules.
What are the pre-requisites for MLDM?
To do MLDM we need 2 things:

1. Data at Multiple Levels of Abstraction

2. An efficient method for Multi-Level Rule Mining (This Paper’s work)
Data at Multiple Levels of Abstraction

We can find Data:

- Implicitly stored in a database
- Provided by Experts or Users
- Data Warehousing and OLAP (Online Analytics Processing)
Data at Multiple Levels of Abstraction

Concept Taxonomies in Databases might look like:
Generalization to Specialization:
(is-a relationship)

Level 1: 
- Diapers, Beer

Level 2: 
- CLOTH, DISPOSABLE
- REGULAR, LITE

Level 3: 
- BUMKINS, KUSHIES, PAMPERS, HUGGIES
- BUDWEISER, SAMUELADAMS, HEINIKEN
Data at Multiple Levels of Abstraction

Generalization to Specialization with Multiple Inheritance:

```
Vehicle
  ↓  ↓
Commuting  Recreational
  ↓  ↓
Car  Bicycle  Snowmobile
```
Data at Multiple Levels of Abstraction

Whole-Part Hierarchies
(is-part-of, has-part)

Computer

MotherBoard
RAM CPU

HardDrive
RWHead Platter
What about Apriori?

What can we try to find rules for these multi-level datasets?

- We can apply Apriori to each level
What about Apriori?

What can we try to find rules for these multi-level datasets?

- We can apply Apriori to each level
- Problems?
What about Apriori?

Problems with Apriori

- Higher levels of abstraction have higher support
- Lower levels have lower support
- Optimum minimal support for all levels?
- Min. Support too high: not enough itemsets in low levels
- Min. Support too low: too many uninteresting rules
Possible Solutions:

- Different minimal support at each level
- Different minimal confidence at each level
- Reduce minimal support as level increases
Progressive Deepening Method

The authors propose a Progressive Deepening method which

- Makes some assumptions about data
- Introduces work around for those who have issues with the assumption
- Is significantly different from other research
Main Assumptions:

- Explore only descendants of frequent items
- If an item is non frequent at one level, none of it's descendants figure in future analysis

What are some problems with this?
Will this eliminate possible interesting rules for itemsets whose ancestors were infrequent?

**Work around**

1. 2 Min. Support values. One absolute cutoff point (normal minisup), one for allowing frequent items to lower levels, called the *Level Passage Threshold LPH*.

2. The LPH can be adjusted by user to allow descendents of sub-frequent items.
How is this different?

- Other research uses same minisup across all levels
- Problems with this?
How is this different?

- Other research uses same minisup across all levels
- Problems with this?
- As said before:
  - Min. Support too high: not enough itemsets in low levels
  - Min. Support too low: too many uninteresting rules
This Study does a few things differently:

- Uses different minisup values at different levels of the hierarchy
- Analyzes different optimization techniques
- Proposes extensions to best methods found
- Implements formal interestingness measures
Each database contains:

1. Item dataset containing item description \( \{ A_i , Description \} \)
2. A transaction dataset \( T \) containing set of transactions
   \( \{ tid , \{ A_p \ldots A_q \} \} * \)

* \( tid \) is transaction identifier (key)
Data Format and Definitions

- **A pattern or itemset** \( A \) is one item \( A_i \) or a set of conjunctive items \( A_i \land \ldots \land A_j \).

- The **support** of a pattern is the number of transactions that contain \( A \) vs the total number of transactions, denoted \( \sigma(A|S) \).

- Confidence \( \phi \) of a rule \( A \Rightarrow B \in S \) is denoted:
  \[
  \phi(A \Rightarrow B) = \frac{\sigma(A \land B)}{\sigma(A)}
  \]
  \( \phi(A \Rightarrow B) \) is the conditional probability of \( B \) occurring given \( A \) has.
Data Format and Definitions

- 2 Thresholds at each level:
  1. minisup ($\sigma'$)
  2. miniconf ($\phi'$)
Data Format and Definitions

GID encoding:

- 112 means Level 1 item 1, level 2 item 1, level 3 item 2.
- For example Hood 1% Milk, might encode to 243 if Milk is the second category of the first level, 1% is the fourth type of milk of the second level, and Hood is the third brand possible brand for an item.
A pattern $A$ is frequent in set $S$ if:
- $\sigma(A) \geq \sigma'$ (Support of $A$ is no less than minimum support for that level)

A rule $A \Rightarrow B$ in $S$ is strong if:
- each ancestor of every item in $A$ and $B$ is frequent at its corresponding level
- $A \land B$ is frequent at the current level
- $\phi(A \Rightarrow B) \geq \phi'$ (The confidence of $A \Rightarrow B$ is no less than the minimum confidence at that level)
What's the point?

So why do we want Strong rules and frequent items?
What’s the point?

- This ensures that the patterns examined at the lower levels arise from itemsets that have a high support at higher levels.
- Strong rules help filter out ‘uninteresting’ rules.
High Level View of the Algorithm

At level 1
1. get frequent itemsets
2. create filtered virtual table

For each other level
1. Generate candidates (Apriori) from frequent itemsets
2. for each transaction
   1. Get subsets and calculate support for generated candidates
   2. Pass to next level if they meet criteria
3. Union all found subsets that have met criteria

Repeat until desired level reached or empty frequent 1-itemset is generated
So how does it work?

Example query: Find multiple level strong association rules for purchase patterns related to *category*, *content*, and *brand*

- Retrieve relevant data from the database relations

<table>
<thead>
<tr>
<th>id</th>
<th>category</th>
<th>content</th>
<th>brand</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>milk</td>
<td>2%</td>
<td>Hood</td>
<td>3.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Merge into generalized table with id’s replaced with id set

<table>
<thead>
<tr>
<th>gid</th>
<th>id</th>
<th>category</th>
<th>content</th>
<th>brand</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>{ 101,102,114 }</td>
<td>milk</td>
<td>2%</td>
<td>Hood</td>
</tr>
<tr>
<td></td>
<td>{ ...,...,...,... }</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
</tbody>
</table>

Han,Fu  Mining Multi-level Association Rules in Large Databases
So how does it work?

Example query: Find multiple level strong association rules for purchase patterns related to *category, content,* and *brand*

- Find frequent patterns and strong rules at highest level. 1-item, 2-item, k-item itemsets may be discovered of the form \{bread, vegetable, milk,\}
- At the next level the process is repeated but the itemsets will be more specific ex: \{2% milk, lettuce, white bread\}
- Repeat previous 2 steps until all levels until no more frequent patterns
L1 = Level 1, L2 = Level 2, L3 = Level 3

food

milk ← L1 → bread

2% ← L2 → chocolate white wheat

Dairyland Foremost ← L3 → oldmills wonder

Han,Fu Mining Multi-level Association Rules in Large Databases
Data Set For Exercise

Let’s do an example to fully understand the algorithm!
Data Set For Exercise

Table 1: Sales Transaction Table

<table>
<thead>
<tr>
<th>Trans id</th>
<th>Bar_code_set</th>
</tr>
</thead>
<tbody>
<tr>
<td>351428</td>
<td>{17325, 92108, \ldots}</td>
</tr>
<tr>
<td>653234</td>
<td>{23423, 56432, \ldots}</td>
</tr>
</tbody>
</table>

Table 2: sales_item (Description) Relation

<table>
<thead>
<tr>
<th>bar_code</th>
<th>category</th>
<th>brand</th>
<th>content</th>
<th>size</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>17325</td>
<td>milk</td>
<td>Foremost</td>
<td>2%</td>
<td>1 Gal</td>
<td>3.31</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Table 3: Generalized sales_item Description Table

<table>
<thead>
<tr>
<th>GID</th>
<th>Barcode_set</th>
<th>Category</th>
<th>Content</th>
<th>brand</th>
</tr>
</thead>
<tbody>
<tr>
<td>112</td>
<td>{17325, 31414, 91265, \ldots}</td>
<td>Milk</td>
<td>2%</td>
<td>Foremost</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Preprocessing

Before running the algorithm we encode the data into the following table using the GID and Transaction Id's.

<table>
<thead>
<tr>
<th>TID</th>
<th>GID encoded Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{ 111, 121, 211, 221 }</td>
</tr>
<tr>
<td>T2</td>
<td>{ 111, 211, 222, 323 }</td>
</tr>
<tr>
<td>T3</td>
<td>{ 112, 122, 222, 323 }</td>
</tr>
<tr>
<td>T4</td>
<td>{ 111, 121 }</td>
</tr>
<tr>
<td>T5</td>
<td>{ 111, 122, 211, 221, 413 }</td>
</tr>
<tr>
<td>T6</td>
<td>{ 211, 323, 524 }</td>
</tr>
</tbody>
</table>
Algorithm Step 1

Created Level 1 Item 1 Table with MiniSup = 4

<table>
<thead>
<tr>
<th>TID</th>
<th>GID encoded Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{ 111, 121, 211, 221 }</td>
</tr>
<tr>
<td>T2</td>
<td>{ 111, 211, 222, 323 }</td>
</tr>
<tr>
<td>T3</td>
<td>{ 112, 122, 222, 323 }</td>
</tr>
<tr>
<td>T4</td>
<td>{ 111, 121 }</td>
</tr>
<tr>
<td>T5</td>
<td>{ 111, 122, 211, 221, 413 }</td>
</tr>
<tr>
<td>T6</td>
<td>{ 211, 323, 524 }</td>
</tr>
</tbody>
</table>

L(1,1) (Level 1 1-itemsets)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1**</td>
<td>5</td>
</tr>
<tr>
<td>2**</td>
<td>5</td>
</tr>
</tbody>
</table>

We can see that 5 transactions support both 1-itemsets
Algorithm Step 2

**L(1,1)**

<table>
<thead>
<tr>
<th>itemset</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1**}</td>
<td>5</td>
</tr>
<tr>
<td>{2**}</td>
<td>5</td>
</tr>
</tbody>
</table>

**L(1,2)**

<table>
<thead>
<tr>
<th>itemset</th>
<th>support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1**, 2**}</td>
<td>4</td>
</tr>
</tbody>
</table>

**Filtered T[2]**

<table>
<thead>
<tr>
<th>TID</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{111, 121, 211, 221}</td>
</tr>
<tr>
<td>T2</td>
<td>{111, 211, 222}</td>
</tr>
<tr>
<td>T3</td>
<td>{112, 122, 221}</td>
</tr>
<tr>
<td>T4</td>
<td>{111, 121}</td>
</tr>
<tr>
<td>T5</td>
<td>{111, 122, 211, 221}</td>
</tr>
<tr>
<td>T6</td>
<td>{211}</td>
</tr>
</tbody>
</table>

The new table is created by filtering the old with respect to L(1,1)
Algorithm Step 3

Level 2 MiniSup = 3

<table>
<thead>
<tr>
<th>TID</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{111, 121, 211, 221}</td>
</tr>
<tr>
<td>T2</td>
<td>{111, 211, 222}</td>
</tr>
<tr>
<td>T3</td>
<td>{112, 122, 221}</td>
</tr>
<tr>
<td>T4</td>
<td>{111, 121}</td>
</tr>
<tr>
<td>T5</td>
<td>{111, 122, 211, 221}</td>
</tr>
<tr>
<td>T6</td>
<td>{211}</td>
</tr>
</tbody>
</table>

Filtered T[2]

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{11*, 12*, 22*}</td>
<td>3</td>
</tr>
<tr>
<td>{11*, 21*, 22*}</td>
<td>3</td>
</tr>
</tbody>
</table>

L(2,3)

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{11*, 12*, 22*}</td>
<td>3</td>
</tr>
<tr>
<td>{11*, 21*, 22*}</td>
<td>3</td>
</tr>
</tbody>
</table>

L(2,1)

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{11*}</td>
<td>5</td>
</tr>
<tr>
<td>{12*}</td>
<td>4</td>
</tr>
<tr>
<td>{21*}</td>
<td>4</td>
</tr>
<tr>
<td>{22*}</td>
<td>4</td>
</tr>
</tbody>
</table>

L(2,2)

<table>
<thead>
<tr>
<th>Itemset</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{11*, 12*}</td>
<td>4</td>
</tr>
<tr>
<td>{11*, 21*}</td>
<td>3</td>
</tr>
<tr>
<td>{11*, 22*}</td>
<td>4</td>
</tr>
<tr>
<td>{12*, 22*}</td>
<td>3</td>
</tr>
<tr>
<td>{21*, 22*}</td>
<td>3</td>
</tr>
</tbody>
</table>
Algorithm Step 3

Level 3 MiniSup = 3

<table>
<thead>
<tr>
<th>TID</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{111, 121, 211, 221}</td>
</tr>
<tr>
<td>T2</td>
<td>{111, 211, 222}</td>
</tr>
<tr>
<td>T3</td>
<td>{112, 122, 221}</td>
</tr>
<tr>
<td>T4</td>
<td>{111, 121}</td>
</tr>
<tr>
<td>T5</td>
<td>{111, 122, 211, 221}</td>
</tr>
<tr>
<td>T6</td>
<td>{211}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L(3,1)</th>
<th>Itemset</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{111}</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>{211}</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>{221}</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L(3,2)</th>
<th>Itemset</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>{111, 211}</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
What counts as interesting?

The paper defines two filters for interesting rules

1. Removal of Redundant Rules
2. Removal of Unnecessary Rules
Redundant Rules

A rule is redundant if

- it can be derived or computed from a higher level rule *
- we assume a relatively uniform distribution

*every item is the same or a higher level item
Redundant Rules

Example:

- $R = milk \Rightarrow bread$ with $\sigma(R) = 12\%$, $\phi(R) = 85\%$
- $R' = chocolate \ milk \Rightarrow bread$ with $\sigma(R') = 1\%$ and $\phi(R') = 84\%$

$R'$ might not be interesting if only 8\% of all milk is chocolate.
Redundant Rules

Formal Definition of Redundant Rule:

A rule $R, A_1 \land A_2 \land \ldots A_n \Rightarrow B_1 \land B_2 \ldots B_m$ is redundant if there is some rule $R', A'_1 \land A'_2, \ldots A'_n \Rightarrow B'_1 \land B'_2 \ldots B'_m$ where every item in $R$ is a descendant or the same in $R'$ and

$$\phi(R) \in [\exp(\phi(R)) - \alpha, \exp(\phi(R)) + \alpha]$$

where

$$\exp(\phi(R)) = (\sigma(B_1)/\sigma(B'_1) \times \cdots \times (\sigma(B_n)/\sigma(B'_n)) \times \phi(R'))$$

and $\alpha$ is a user defined constant.

In english: If the confidence of a rule falls within a certain range and its items are shared in other rules, it is Redundant.
Redundant Rules

Applying Redundant Rule reduction cuts Strong Rules by 40-70%
A rule is unnecessary if
- it does not differ significantly from a simpler rule.
Unnecessary Rules

Example:

- 80% of Customers who buy milk also buy bread
- 80% of Customers who buy milk and butter also buy bread

The extra information doesn’t really tell us anything new.
Unnecessary Rules

Formal Definition: A rule $R, A \land C \Rightarrow B$ is unnecessary if there is a rule $R', A \Rightarrow B$ and $\phi(R) \in [\phi(R') - \beta, \phi(R') + \beta]$ where $\beta$ is a user defined constant. $A, B, C$ are itemsets and $C$ is not empty.
Unnecessary Rules

Applying Unnecessary rule reduction cuts Strong rules by 20-50%
List of Variations

Variations on ML_T2L1 (Original Algorithm)

1. ML_T1LA
2. ML_TML1
3. ML_T2LA

(There are also two extensions on T2LA and T2L1 but they are beyond the scope of this presentation)
Variations:

- Uses only one encoded table $T[1]$
- Computes support for all levels of hierarchy with one scan

Pros? Cons?
Pros:

- Avoids generation of new transaction table
- Total number of scans = k for largest k-itemset
Cons:
- Scanning \( T[1] \) scans infrequent items which is wasteful
- Large space for all subsets, page swapping possibly needed
Variation:

- Instead of just $T[1]$ and $T[2]$. We have $T[1], T[2], \ldots, T[\text{max l} + 1]$
- Generate each $T[l]$ in parallel.
Pros:

- Newly filtered tables will cut down amount of processing at each level if some data is frequent
Cons:

- Small number of items filtered out at processing each level. Will make the algorithm run for a long time.
ML_T2LA

Variation:
- Uses T[1] and T[2] (like original) but optimizes like ML_T1LA
- Calculates support with one scan of T[2]
- Scans k-1 times per generation of all k-itemsets
Pros:

- Scans $T[2]$ $k$-1 times total for generation of all $k$-itemsets.
- Potentially efficient due to filtering of $T[1]$
Cons:

- The efficiency of this algorithm depends on the data being able to be filtered.
According to the book, ML_T1LA, is the best or second best. However I believe to be a typo, the charts indicate that ML_T2LA is the best most of the time.
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According to the book, ML_T1LA, is the best or second best. However I believe to be a typo, the charts indicate that ML_T2LA is the best most of the time.
Conclusions

The authors

- Extended association rules from single to multi level
- A top-down progressive deepening technique was developed for finding such rules
- Filtering of uninteresting rules was formally defined in two ways.
Future work

The future may hold

- Developing efficient algorithms for mining multi-level sequential patterns
- Mining multiple level correlations in databases
- Cross level associations
- More interestingness measures of rules
Question 1

What is a major drawback to multiple level data mining using the same minimal support at all levels of a concept hierarchy?

Large Support exists at higher levels of abstraction, and smaller support at lower levels. To find strong rules in the deeper levels we must relax support at higher levels, which can result in uninteresting rules at higher levels. It is hard to determine an optimal minimal support for all levels.
Question 2

What are the 2 pre-requisites to performing multiple-level association rule mining?

To explore multiple level association rule mining one needs to provide:

1. Data at multiple levels of abstraction
2. Efficient methods for multiple level rule mining
Question 3

- Give an example of a multiple level association rule
- At a high level in the hierarchy one may have a general rule like 80% of people who buy cereal also buy milk, at a lower level the rule becomes more specific like 25% of people who buy cheerios buy almond milk from silk.